

Market-Based Executive Compensation Contract under Endogenous Information Acquisition*

Guangsug Hahn[†] Joon Yeop Kwon[‡]

Abstract The study investigates how a publicly traded firm's liquidation value and stock price are used in an executive compensation contract when information acquisition in the asset market is endogenized. If the inside owner offers market-based compensation contract to the risk-averse manager, the inside owner expects higher utility than when stock prices are excluded from the contract. If information cost displays an intermediate value, changes in the exogenous parameters generate the direct effect and the indirect effect via the information market. Finally, we find that the market-based compensation contract contributes to the increase of social welfare.

Keywords market-based executive compensation; information acquisition

JEL Classification G30, D86

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[†]Division of Humanities and Social Sciences, POSTECH, Korea, econhahn@postech.ac.kr

[‡]Corresponding Author, School of Business Administration, Kyungpook National University, Korea, joonyeop.kwon@knu.ac.kr, +82-53-950-5449.

1. INTRODUCTION

During recent decades, executive compensation for top management has drastically increased in the United States compared with economic growth.¹ Extremely high managerial payments cause debates about the optimal managerial incentive schemes. Firm owners want managers to take actions that maximize firm value. However, since owners cannot perfectly observe managerial activities, managers may pursue their own private interests. Earlier studies such as Mirrlees (1976) and Holmström (1979) discuss executive compensation models based on firms' liquidation values for risk-sharing purposes. On the contrary, Hayek (1945) emphasizes that the price system plays a role in transmitting information about economic conditions. From this point of view, Holmström and Tirole (1993) and Baiman and Verrecchia (1995) show that incorporating stock prices into executive compensation is helpful for monitoring managerial performance.² In the empirical study of Jensen and Murphy (1990), it is shown that executive compensation tends to increase stock prices.

This study investigates the effect of incorporating the stock price into an executive compensation contract when the information acquisition of rational traders is endogenized. To do this, we incorporate the standard principal-agent problem (e.g., Holmström, 1979) into Grossman and Stiglitz's (1980) asset pricing model with asymmetric information. In our model, risk-neutral inside owners who own large shares of the firm's equity offer a compensation contract to a risk-averse manager who will operate the firm. In addition to a fixed wage, the manager can earn a bonus payment that depends on the firm's liquidation value and stock price. After accepting the contract, the manager makes unobservable efforts at his own cost and the firm's liquidation value is affected by these efforts. In the asset market, on the contrary, rational traders decide whether to purchase information about the firm's fundamental value, which is realized by the effort level chosen by the manager. Based on individual information, rational traders

¹See Bebchuk and Grinstein (2005).

²See also Kim and Suh (1993), Kang and Liu (2010), and Calcagno and Heider (2014) among others.

and liquidity traders take positions and the stock price is determined.

In this study, we examine the role of the stock price in executive compensation contracts. We find that inside owners can be better off by incorporating the stock price into the compensation contract. Under the market-based compensation contract, the manager makes more efforts for a higher payoff, causing a higher expected liquidation value for the firm compared with the case in which the contract only depends on its liquidation value. However, since the increase in the firm's expected liquidation value dominates that in the expected compensation to the manager, inside owners can increase their expected profit. This result is consistent with those of Holmström and Tirole (1993), Baiman and Verrecchia (1995), and Kang and Liu (2010), who show the monitoring role of the stock price in executive compensation contracts.³ In our model, however, an increase in the firm's liquidation value or the stock price leads to an increase in expected managerial income as in Holmström and Tirole (1993), while in Baiman and Verrecchia (1995) and Kang and Liu (2010), an increase in the firm's liquidation value lowers the manager's expected compensation. Further, the role of the stock price in the compensation contract is effective only when the manager is risk-averse. If inside owners make a contract with a risk-neutral manager, the manager chooses the same effort level, which leads to the same expected liquidation value regardless of whether the stock price is incorporated. Consequently, they cannot be better off from the market-based compensation contract.

We also investigate the relationship between the optimal contract and exogenous variables such as rational traders' degree of risk aversion, the information cost, market liquidity, and the variance in the firm's fundamental value. This study focuses on how the relative importance of the liquidation value to the stock price in the contract is affected by changes in the exogenous variables and interprets the effects in terms of the informativeness of the liquidation value and the stock price. Since we endogenize the information acquisition of rational traders in the asset market, we should consider both the direct and the indirect effects generated by the movement of these variables on the optimal contract. If the information cost takes an intermediate value, a proportion of rational traders be-

³See also Kim and Suh (1993).

comes informed by purchasing information about the firm's fundamental value. Then, a change in the exogenous variables makes the proportion of informed traders move and this generates both indirect and direct effects on the optimal contract. On the contrary, if the information cost is sufficiently high or low, all rational traders choose to be uninformed or informed, respectively, and the proportion of informed traders remains unchanged, while the exogenous variables move. Then, the changes in the exogenous variables generate only direct effects on the optimal contract.

Further, we show that social welfare is enhanced by the market-based executive compensation contract. Similar to Kang and Liu (2010), social welfare is measured by the sum of the ex ante expected utilities of the inside owners, manager, and rational traders. Indeed, the manager's expected utility is not affected by the contract scheme and remains unchanged at his reservation value since his individual rationality constraints are always binding. Rational traders' ex ante expected utilities are also independent of the managerial contract. Since an increase in managerial efforts raises the firm's expected liquidation value and the stock price by the same amount, the manager's effort level is canceled out in rational traders' ex ante expected utilities. On the contrary, inside owners can be better off when they offer a market-based compensation contract to a risk-averse manager than otherwise since an increase in the firm's liquidation value due to a higher managerial effort dominates an increase in the expected compensation to the manager. Therefore, the overall effect increases social welfare by incorporating the stock price into the contract.

This study is closely related to the model of Holmström and Tirole (1993), who also examine the value of the stock price in executive compensation contracts. They adopt the asset pricing model of Kyle (1985) in which competitive market makers set stock prices after observing the aggregate order flows of an informed trader and liquidity traders. They show that the stock price plays a monitoring role for managerial performance and that a change in the exogenous variables generates an indirect effect on the optimal contract via the informed trader's choice of information quality. However, their model does not take into account the endogenous information acquisition of ex ante identical ra-

tional traders. They simply assume an inborn informed trader who can control the quality of information. In our model, the information market equilibrium is determined when all rational traders have the same ex ante expected utility. We show that a change in the exogenous variables affects the proportion of informed traders and thus changes the optimal contract offered by inside owners.

This study is also related to Kang and Liu (2010), who examine the effect of the endogenous information acquisition of ex ante identical rational traders on the optimal contract. They show that the optimal contract is subject to the information acquisition of rational traders as in our study. By adopting the framework of Kyle (1985), however, they do not allow the participation of uninformed rational traders. As long as the information cost is finite, in their model, informed traders take positions and a change in the exogenous variables affects the number of informed traders. Thus, the optimal contract is always indirectly affected by the information market equilibrium. On the contrary, our model encompasses all possible proportions of informed (or uninformed) traders since all rational traders participate in trading based on individual information. Our model can thus explain the case in which all rational traders are informed or uninformed and thus the optimal contract is not influenced by the information market equilibrium. In particular, we find that a change in the variance in the firm's fundamental value differently affects the optimal contract when the indirect effect occurs and otherwise.

The rest of the paper is organized as follows. In Section 2, we introduce the model of our linear executive compensation contract when asymmetric information among traders exists in the asset market. The asset market equilibrium is derived in Section 3. In Section 4, we derive and characterize the optimal contract between the inside owners and the manager. The effect of incorporating the stock price into the contract on social welfare is examined in Section 5. Concluding remarks are given in Section 6. All the proofs are relegated to the Appendix.

2. THE MODEL

We consider an economy in which there are three dates, indexed by $t = 0, 1, 2$. Risk-neutral inside owners hold a large proportion $\delta \in [0, 1]$ of the firm's whole equity share 1 until the final date ($t = 2$).⁴ For simplicity, we henceforth assume that the inside owners are represented by a single person. At the initial date ($t = 0$), a firm is established and its stocks are issued. The inside owner (she) offers the risk-averse manager (him) a compensation contract, which is based on the firm's liquidation value and stock price. The manager chooses his effort level, which affects the liquidation value and stock price. At $t = 1$, rational traders decide whether to purchase information about the firm's fundamental value and then make portfolio choices. The stocks are publicly traded and the stock price is determined. At $t = 2$, the liquidation value is realized and the inside owner, manager, and traders are paid.

2.1. EXECUTIVE COMPENSATION

At $t = 0$, the inside owner offers an incentive contract I to the manager, which includes the compensation for managerial performance measured by liquidation value v and stock price p :

$$I = a_0 + a_1 v + a_2 p,$$

where a_0 represents a fixed wage, and a_1 and a_2 denote the weights on performance measures v and p , respectively. Accepting the contract, the manager chooses effort level e at $t = 0$, which is unobservable to the inside owner and outside traders. His effort level e involves monetary cost $h(e) = ke^2/2$, where $1/k$ measures the efficiency of the managerial effort. His effort choice affects the firm's fundamental value θ realized at $t = 1$, which is given by the sum of managerial effort level e chosen by the manager and external fluctuation η beyond the manager's control: $\theta = e + \eta$, where η has a normal distribution with mean zero and variance σ_η^2 .

⁴Unlike Holmström and Tirole (1993), δ is exogenously given for analytical convenience.

We assume that the manager is prohibited from trading the stock. At $t = 2$, liquidation value v is realized. This liquidation value consists of fundamental value θ and noise ε : $v = \theta + \varepsilon$, where ε has a normal distribution with mean zero and variance σ_ε^2 . We assume that fundamental value θ is more volatile than noise ε : $\sigma_\eta^2 > \sigma_\varepsilon^2$.

The inside owner and manager are paid at $t = 2$. The risk-neutral inside owner takes her own share δ of realized liquidation value v net of managerial compensation I . Thus, her utility is given by $u_o(w) = w$, where $w = \delta(v - I)$. We assume that the manager has a CARA utility function with risk aversion coefficient $\gamma_m > 0$: $u_m(w) = -\exp(-\gamma_m w)$, where $w = I - h(e)$.

2.2. ASSET MARKET

Two securities are traded in the asset market at $t = 1$: a risky stock issued by the firm and a risk-free bond. The prices of the stock and bond are given by p and 1, respectively. The bond is assumed to be a numeraire. Rational trader τ invests his initial wealth w_0 in b_τ shares of the bond and x_τ shares of the stock with the budget constraint $b_\tau + px_\tau = w_0$. At $t = 2$, the stock yields liquidation value v . For analytical convenience, we assume that rational traders in the stock market consider the liquidation value itself instead of the liquidation value net of executive compensation when they make investment decisions.⁵

There is a continuum of rational traders, who are utility maximizers, indexed by τ in the interval $[0, 1]$. All rational traders are ex ante identical and have a CARA utility function with risk aversion coefficient $\gamma > 0$: $u(w) = -\exp(-\gamma w)$. The manager makes effort e at $t = 0$ and then the firm's fundamental value θ is realized at $t = 1$, at which point rational traders decide whether to purchase information about θ at cost c . A rational trader who pays (does not pay) c for information about θ is called an informed (uninformed) trader. Informed traders observe a realization of θ in addition to a realization of p , while uninformed traders only observe p when $t = 1$. Thus, at $t = 2$, informed traders' wealth

⁵See also Baiman and Verrecchia (1995), Milbourn (2003), and Kang and Liu (2005) for this convention.

becomes $w_i = w_0 - c + (v - p)x_i$ and uninformed traders' wealth becomes $w_u = w_0 + (v - p)x_u$. Let $\lambda \in [0, 1]$ denote the proportion of informed traders among rational traders.

There exist liquidity traders who are not utility maximizers and they participate in stock trading for exogenous reasons. Their demand is denoted by z , which is normally distributed with mean zero and variance σ_z^2 . As in Holmström and Tirole (1993), we measure market liquidity by σ_z^2 : the higher σ_z^2 , the greater is market liquidity. It is assumed that both informed and uninformed traders have rational expectations in that they understand the functional relationship \tilde{p} between p and (θ, z) .

Figure 1 illustrates the sequence of events.

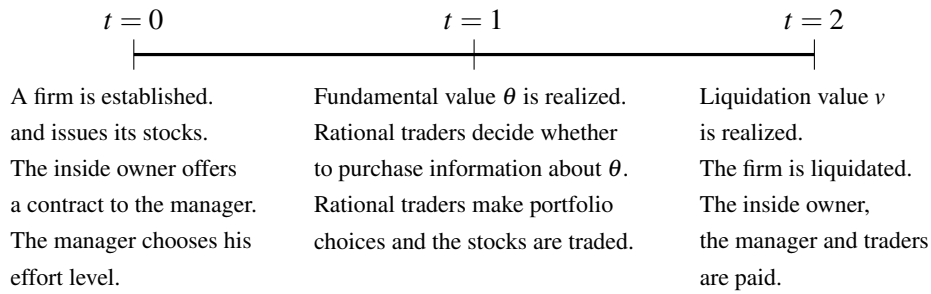


Figure 1: Sequence of events

3. ASSET MARKET EQUILIBRIUM

For the asset market equilibrium, we adopt the notion of the rational expectations equilibrium of Grossman and Stiglitz (1980). The asset market consists of an information market in the first stage and a stock market in the second stage. In the information market, ex ante identical rational traders decide whether to purchase costly information about the firm's fundamental value θ . In the stock market,

rational traders submit orders to buy or sell stocks and the equilibrium stock price is determined. To find the overall asset market equilibrium, we solve the problem through backward induction. That is, we first find the stock market equilibrium for a given proportion λ of informed traders. By determining equilibrium λ in the information market, we derive and characterize the overall equilibrium.

3.1. STOCK MARKET EQUILIBRIUM

First, we derive the equilibrium stock price for a given proportion λ of informed traders. For the optimal portfolio choice, informed trader i solves

$$\max_{x_i} \mathbb{E}[-\exp(-\gamma[w_0 - c + (\tilde{v} - p)x_i]) | (\tilde{p}, \tilde{\theta}) = (p, \theta)]$$

and his demand for the stock is given by

$$x_i(p, \theta) = \frac{\theta - p}{\gamma\sigma_\varepsilon^2}.$$

Similarly, uninformed trader u solves

$$\max_{x_u} \mathbb{E}[-\exp(-\gamma[w_0 + (\tilde{v} - p)x_u]) | \tilde{p} = p]$$

and his demand for the stock is given by

$$x_u(p, \tilde{p}) = \frac{\mathbb{E}[\tilde{v} | \tilde{p} = p] - p}{\gamma\text{Var}[\tilde{v} | \tilde{p} = p]}.$$

Equilibrium stock price function \tilde{p} satisfies the market-clearing condition: for every $p = \tilde{p}(\theta, z)$,⁶

$$\lambda x_i(p, \theta) + (1 - \lambda)x_u(p, \tilde{p}) + z = 1 - \delta. \quad (1)$$

Following Grossman and Stiglitz (1980), we define compound signal function $\tilde{s}: (\theta, z) \mapsto s$, which encapsulates θ and z :

$$\tilde{s}(\theta, z) = \begin{cases} \theta - \frac{\gamma\sigma_\varepsilon^2}{\lambda}(1 - \delta - z) & \text{if } \lambda \in (0, 1], \\ -(1 - \delta - z) & \text{if } \lambda = 0. \end{cases}$$

⁶Since we assume that the firm's whole equity share is 1 and consider a continuum of rational traders, mean demand x_τ is a proportion of the firm's equity.

Clearly, \tilde{s} is normally distributed with mean $e - \gamma\sigma_\varepsilon^2(1 - \delta)/\lambda$ and variance $\sigma_s^2 \equiv \sigma_\eta^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2/\lambda^2$ if $\lambda \in (0, 1]$ and is normally distributed with mean $-(1 - \delta)$ and variance σ_z^2 if $\lambda = 0$. We define equilibrium price function $P : \mathbb{R} \rightarrow \mathbb{R}$ by $P(\tilde{s}(\theta, z)) := \tilde{p}(\theta, z)$ and conjecture that P strictly increases in signal s , which is verified by Proposition 1.

Proposition 1. *Let e^* be the equilibrium effort level of the manager. For a given $\lambda \in [0, 1]$, equilibrium price function P is given by*

$$P(s) = \begin{cases} (1 - \alpha)e^* + \alpha s & \text{if } \lambda \in (0, 1], \\ e^* + \gamma(\sigma_\eta^2 + \sigma_\varepsilon^2)s & \text{if } \lambda = 0, \end{cases} \quad (2)$$

where

$$\alpha = \frac{\lambda^2\sigma_\eta^2 + \lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \lambda\gamma^2\sigma_\varepsilon^4\sigma_z^2}{\lambda^2\sigma_\eta^2 + \lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2}.$$

3.2. INFORMATION MARKET EQUILIBRIUM

We have derived the stock market equilibrium for a given proportion λ of informed traders. In the information market, λ is endogenously determined such that all rational traders have the same (ex ante) expected utility. If the expected utility $\mathbb{E}[u(w_i)]$ of would-be informed traders is higher than that of would-be uninformed traders $\mathbb{E}[u(w_u)]$ (i.e., $\mathbb{E}[u(w_i)] > \mathbb{E}[u(w_u)]$), would-be uninformed traders will purchase information about θ at cost c and thus the proportion λ of informed traders will increase. On the contrary, if uninformed traders' expected utility is higher than that of informed ones (i.e., $\mathbb{E}[u(w_i)] < \mathbb{E}[u(w_u)]$), would-be informed traders decide to remain uninformed and thus λ will decrease.

Before the information market is opened, the expected utilities of would-be informed traders and would-be uninformed traders are given by, respectively,

$$\mathbb{E}[u(w_i)] = e^{\gamma c} u(w_0) \sqrt{\frac{\kappa}{1 + \nu}} \quad \text{and} \quad \mathbb{E}[u(w_u)] = u(w_0) \sqrt{\frac{1}{1 + \nu}}, \quad (3)$$

where

$$\begin{aligned}\kappa &= \frac{\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2}{\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2}, \\ \nu &= \frac{\gamma^4 \sigma_\varepsilon^6 \sigma_z^4 (\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2)}{(\lambda^2 \sigma_\eta^2 + \lambda \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2)^2}.\end{aligned}$$

From (3), the ratio of expected utility between would-be informed and uninformed traders is given by

$$\varphi(\lambda) := \frac{\mathbb{E}[u(w_i)]}{\mathbb{E}[u(w_u)]} = e^{\gamma c \sqrt{\kappa}}. \quad (4)$$

Since

$$\frac{\partial \kappa}{\partial \lambda} = \frac{2\lambda \gamma^2 \sigma_\eta^4 \sigma_\varepsilon^2 \sigma_z^2}{(\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2)^2} > 0, \quad (5)$$

ratio function φ increases in λ . As the proportion λ of informed traders increases, the relative expected utility of informed traders with respect to uninformed ones decreases.⁷ As Grossman and Stiglitz (1980) point out, more traders become informed, more of their information is revealed to uninformed traders through stock prices. That is, uninformed traders can improve their portfolio by free riding on informed traders' costly information. Thus, as more rational traders acquire information about θ , the others have less incentive to purchase it and information acquisition exhibits *strategic substitutability*.

As a consequence, we have the following information market equilibria:

$$\begin{cases} \lambda = 0 & \text{if } \varphi(0) \geq 1, \\ \lambda = 1 & \text{if } \varphi(1) \leq 1. \\ \lambda \in (0, 1) & \text{if } \varphi(0) < 1 < \varphi(1). \end{cases}$$

Then, it is straightforward to obtain Proposition 2.

Proposition 2. *The following hold.*

⁷Recall that we assume a negative utility function of rational traders.

(1) If information cost c is sufficiently high such that

$$c \geq \frac{1}{2\gamma} \ln \left(1 + \frac{\sigma_\eta^2}{\sigma_\varepsilon^2} \right) \equiv \bar{c}, \quad (6)$$

all rational traders remain uninformed, i.e., $\lambda = 0$.

(2) If information cost c is sufficiently low such that

$$0 < c \leq \frac{1}{2\gamma} \ln \left(1 + \frac{\gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2}{\sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2} \right) \equiv \underline{c}, \quad (7)$$

all rational traders become informed, i.e., $\lambda = 1$.

(3) If information cost c takes intermediate value such that

$$\underline{c} < c < \bar{c}, \quad (8)$$

a proportion of rational traders become informed, i.e., $\lambda \in (0, 1)$.

Threshold information costs \underline{c} and \bar{c} in Proposition 2 determine the types of information market equilibrium. If the cost lies on (\underline{c}, \bar{c}) , we have the interior solution in which informed and uninformed traders coexist. If not, we have the corner solution, in which either informed or uninformed traders exist. This difference in types of equilibrium play crucial role when we characterize the equilibrium contract in Section 4.

Suppose that the information market has an interior equilibrium, i.e., $\lambda \in (0, 1)$ such that $\varphi(\lambda) = 1$. This implies that if information cost c increases, a smaller proportion of rational traders becomes informed to keep $\varphi(\lambda) = 1$ from (5). On the other hand, if c is fixed, then κ also remains unchanged. Note that κ can be rewritten as

$$\kappa = \left(1 + \frac{\gamma^2 \sigma_\varepsilon^2 \sigma_z^2}{\zeta_\eta} \right)^{-1}$$

where

$$\zeta_\eta \equiv \lambda^2 + \frac{\gamma^2 \sigma_\varepsilon^4 \sigma_z^2}{\sigma_\eta^2}. \quad (9)$$

Since variance σ_η^2 of the firm's fundamental value is only contained in ζ_η , an increase in σ_η^2 induces an increase in λ to have κ unchanged. We also have

$$\kappa = \left(1 + \frac{\gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2}{\zeta_z \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^4} \right)^{-1}$$

where

$$\zeta_z \equiv \frac{\lambda^2}{\sigma_z^2}. \quad (10)$$

From a similar argument, an increase in market liquidity σ_z^2 leads to an increase in λ . We summarize these properties of the interior equilibrium of the information market below.⁸

Proposition 3. *Suppose that information cost c satisfies (8). In the information market equilibrium, the following hold.*

- (1) *If information cost c increases satisfying (8), the proportion λ of informed traders decreases.*
- (2) *If market liquidity σ_z^2 or variance σ_η^2 of the firm's fundamental value increases satisfying (8), the proportion λ of informed traders increases.*

Rational traders' risk preferences also affect information acquisition. Rational traders' risk aversion coefficient γ affects both $e^{\gamma c}$ and $\sqrt{\kappa}$. Thus, to have $\varphi(\lambda) = 1$, a change in $e^{\gamma c}$ is exactly offset by that in $\sqrt{\kappa}$ when γ moves. We summarize the effect of a change in γ on λ in Proposition 4.

Proposition 4. *Let $c^* = \max\{c', \underline{c}\}$ where*

$$c' = \frac{\sigma_\varepsilon^2}{\gamma \sigma_\eta^2} \left(\sqrt{1 + \frac{\sigma_\eta^2}{\sigma_\varepsilon^2}} - 1 \right)^2.$$

Suppose that information cost c takes an intermediate value such that

$$c \in (c^*, \bar{c}) \equiv C. \quad (11)$$

As a rational trader's risk aversion coefficient γ increases, the proportion λ of informed traders increases.

⁸See also Grossman and Stiglitz (1980) for similar results.

If the information cost lies in sophisticated region C , as rational traders are more risk-averse, fewer traders choose to purchase information about the firm's fundamental value.

3.3. INFORMATIVENESS OF THE STOCK PRICE AND LIQUIDATION VALUE

From the perspective of the inside owner, the stock price contains information about the managerial effort level. From (2), we know that a higher stock price is expected as the manager makes more efforts. However, since the stock price also contains noise terms outside managerial control, the inside owner cannot extract precise information about managerial efforts from the stock price. Thus, as the stock price becomes more volatile due to the noise, the inside owner faces more difficulty extracting information about the managerial effort level from the stock price. Variance σ_p^2 in the stock price is given by

$$\sigma_p^2 = \frac{(\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2) (\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2)^2}{(\lambda^2 \sigma_\eta^2 + \lambda \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2)^2}, \quad \lambda \in [0, 1]. \quad (12)$$

We define the informativeness of the stock price (*price informativeness*) based on the precision τ_p of the stock price, given by⁹

$$\tau_p = \frac{1}{\sigma_p^2}.$$

Liquidation value v also transmits information about managerial efforts. Similar to the stock price, as v fluctuates more, it contains less information about managerial efforts. We define the informativeness of the liquidation value (*value informativeness*) based on the precision τ_v of the liquidation value:

$$\tau_v = \frac{1}{\sigma_\eta^2 + \sigma_\varepsilon^2}. \quad (13)$$

⁹The concept of price informativeness follows the notion of Vives (1995). However, it is different from that of Grossman and Stiglitz (1980), who measure the correlation between the stock price and fundamental value.

Proposition 5. *Suppose that information cost c satisfies (8). The following hold.¹⁰*

(1) *Assume that rational traders are sufficiently risk-averse to the extent that*

$$\gamma > \frac{1}{\sqrt{\sigma_\varepsilon \sigma_z}}. \quad (14)$$

If information cost c increases satisfying (8), then price informativeness τ_p decreases.

(2) *Suppose that (14) holds. If rational traders' risk aversion coefficient γ increases satisfying (11), then price informativeness τ_p decreases.*

(3) *If market liquidity σ_z^2 increases satisfying (8), then price informativeness τ_p decreases.*

(4) *If variance σ_η^2 of the firm's fundamental value increases, then value informativeness τ_v decreases.*

A change in information cost c indirectly affects τ_p via information acquisition without any direct effect:

$$\frac{d\tau_p}{dc} = \underbrace{\frac{\partial \tau_p}{\partial \lambda} \frac{\partial \lambda}{\partial c}}_{\text{indirect effect of } c}.$$

As information becomes costlier to acquire, fewer traders choose to be informed. On the other hand, an increase in the proportion λ of informed traders does not always increase price informativeness τ_p .¹¹ However, if rational traders are sufficiently risk-averse, the stock price contains more information about managerial effort as more traders become informed. Therefore, an increase in the information cost makes the stock price less informative when rational traders are sufficiently risk-averse.

A change in rational traders' risk aversion coefficient γ has both a direct effect and an indirect effect via information acquisition on price informativeness

¹⁰Since the effect of σ_η^2 on τ_p is not unambiguously determined, we omit any analysis about it.

¹¹For a similar result, see Black and Tonks (1992), who observe the non-monotonic relationship between λ and σ_p^2 in the setup of Grossman and Stiglitz (1980) excluding information acquisition.

τ_p :

$$\frac{d\tau_p}{d\gamma} = \underbrace{\frac{\partial \tau_p}{\partial \gamma}}_{\text{direct effect of } \gamma} + \underbrace{\frac{\partial \tau_p}{\partial \lambda} \frac{\partial \lambda}{\partial \gamma}}_{\text{indirect effect of } \gamma} .$$

The direct effect decreases τ_p from (A.28). Furthermore, the indirect effect decreases τ_p . This is because as rational traders become more risk-averse, fewer traders purchase information about θ when c lies in C from Proposition 11 and as more traders becomes informed, price informativeness increases when (14) holds from (A.27). Therefore, the stock price contains less information about managerial efforts as rational traders become more risk-averse when they are sufficiently risk-averse and the information cost lies in the sophisticated intermediate region.

Similarly, a change in market liquidity σ_z^2 has both a direct effect and an indirect effect via information acquisition on τ_p :

$$\frac{d\tau_p}{d\sigma_z^2} = \underbrace{\frac{\partial \tau_p}{\partial \sigma_z^2}}_{\text{direct effect of } \sigma_z^2} + \underbrace{\frac{\partial \tau_p}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma_z^2}}_{\text{indirect effect of } \sigma_z^2} .$$

The signs of both the direct and the indirect effects are not clearly determined, but the overall effect always lessens τ_p . Therefore, as the asset market becomes more liquid, the stock price transmits less information about managerial efforts.

Although value informativeness τ_v is not affected by information cost c , market liquidity σ_z^2 , or risk aversion coefficient γ , it is affected by variance σ_η^2 of the firm's fundamental value. As fundamental value θ becomes more volatile, the monitoring role of v for managerial efforts lessens.

4. EQUILIBRIUM CONTRACT

This section is devoted to deriving and analyzing the optimal compensation contract. To maximize her utility, the inside owner designs the contract, which will be accepted by the manager. Since the inside owner takes the realized liquidation value net of managerial compensation, it is easy to believe that she may achieve

her goal if she simply asks the manager to maximize the liquidation value. However, incorporating the stock price into the contract may affect the managerial effort level, which in turn impacts both the liquidation value and managerial compensation. For the inside owner, the overall effect of incorporating the stock price into the contract is difficult to predict. Before we derive the optimal incentive contract of our model, we consider the optimal contract depending only on the liquidation value, which we call the *benchmark*. By comparing both contracts, we examine the effect of incorporating the stock price into the incentive contract. We assume that the reservation utility of the manager equals one.

4.1. BENCHMARK CASE: NON-MARKET-BASED COMPENSATION

As the benchmark case, we suppose that the inside owners consider compensation contract I which adopts only liquidation value v as a performance measure:

$$I = a_0 + a_1 v$$

where a_0 is the fixed payment and a_1 is the weight on the liquidation value. Since the inside owner is risk-neutral, her utility function is given by

$$u_o(\delta(v - I)) = \delta(v - I).$$

The inside owner solves the following problem considering individual rationality and incentive compatibility constraints for the manager:¹²

$$\begin{aligned} & \max_{a_0, a_1, e} \mathbb{E}[u_o(\delta(v - I))] \\ & s.t. \quad \mathbb{E}[I] - \frac{\gamma_m}{2} \text{Var}[I] - \frac{1}{2} k e^2 \geq 0, \\ & \quad e = \arg\max_{e'} \mathbb{E}[I] - \frac{\gamma_m}{2} \text{Var}[I] - \frac{1}{2} k (e')^2. \end{aligned}$$

Then, we obtain the following equilibrium contract.

¹²We use the certainty equivalent of the manager's utility in both constraints.

Proposition 6. *In the equilibrium, the optimal compensation contract is given by*

$$\begin{aligned}\hat{a}_0 &= \frac{-1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2)}{2k(1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))^2} \\ \hat{a}_1 &= \frac{1}{1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2)} > 0,\end{aligned}$$

and the equilibrium effort level is

$$\hat{e} = \frac{\hat{a}_1}{k} = \frac{1}{k(1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))}. \quad (15)$$

The weight \hat{a}_1 on the liquidation value is higher than zero. Thus, the manager has an incentive to work to increase the liquidation value. Since the individual rationality constraint is binding in the equilibrium, the manager's expected utility $\mathbb{E}[\hat{u}_m]$ becomes one. Then, we have

$$\mathbb{E}[\hat{I}] = \frac{\gamma_m}{2} \text{Var}[\hat{I}] + \frac{1}{2}k\hat{e}^2 = \frac{\hat{e}}{2}. \quad (16)$$

Since

$$\mathbb{E}[\hat{v}] = \hat{e},$$

we have

$$\mathbb{E}[\hat{u}_o] = \delta\mathbb{E}[\hat{v} - \hat{I}] = \frac{\delta\hat{e}}{2}. \quad (17)$$

Thus, expected managerial income $\mathbb{E}[\hat{I}]$, expected liquidation value $\mathbb{E}[\hat{v}]$, and the inside owner's expected utility $\mathbb{E}[\hat{u}_o]$ have positive linear relationships with optimal effort level \hat{e} .

The managerial effort level and optimal contract depend on the manager's characteristics such as degree γ_m of risk aversion and degree k of effort inefficiency. Proposition 6 shows that as the manager is more risk-averse or his effort becomes costlier, the inside owner offers a contract that puts a higher fixed income \hat{a}_0 and a lower weight \hat{a}_1 on the liquidation value. According to (15), an

increase in γ_m decreases managerial effort level \hat{e} indirectly via \hat{a}_1 and an increase in k decreases \hat{e} both directly and indirectly via \hat{a}_1 . This implies a lower expected income $\mathbb{E}[\hat{I}]$ of the manager from (16) and a lower expected liquidation value $\mathbb{E}[\hat{v}]$. From (17), we know that the decrease in $\mathbb{E}[\hat{v}]$ exceeds that in $\mathbb{E}[\hat{I}]$ and thus the inside owner's expected utility $\mathbb{E}[\hat{u}_0]$ decreases. Therefore, the inside owner prefers to make a compensation contract with a manager who has as low risk aversion and effort inefficiency as possible. The arguments above are summarized in the following corollary.

Corollary 7. *In the benchmark, if the manager's degree γ_m of risk aversion or the manager's degree k of effort inefficiency increases, the following hold.*

- (1) *Fixed income \hat{a}_0 increases and weight \hat{a}_1 on the liquidation value decreases.*
- (2) *The manager's optimal effort level \hat{e} , the manager's expected income $\mathbb{E}[\hat{I}]$, the expected liquidation value $\mathbb{E}[\hat{v}]$, and the inside owner's expected utility $\mathbb{E}[\hat{u}_0]$ decrease.*

4.2. MARKET-BASED COMPENSATION CONTRACT

We now return to our original model in which the stock price is incorporated into the contract, given by

$$I = a_0 + a_1 v + a_2 p.$$

The inside owner maximizes her expected utility under the conditions of individual rationality and incentive compatibility for the manager:

$$\begin{aligned} & \max_{a_0, a_1, a_2, e} \mathbb{E}[u_o(\delta(v - I))] \\ & \text{s.t. } \mathbb{E}[I] - \frac{\gamma_m}{2} \text{Var}[I] - \frac{1}{2} k e^2 \geq 0, \\ & e = \operatorname{argmax}_{e' \in [0, \bar{e}]} \mathbb{E}[I] - \frac{\gamma_m}{2} \text{Var}[I] - \frac{1}{2} k (e')^2 \end{aligned}$$

where \bar{e} is sufficiently high. We obtain the equilibrium contract as follows.

Theorem 8. *In the equilibrium, the optimal compensation contract is given by*

$$\begin{aligned} a_0^* &= -\frac{y + 2(1 - \delta) \left((1 - \alpha) \sigma_\eta^2 + \sigma_\varepsilon^2 \right) kx \lambda \gamma \sigma_\varepsilon^2}{2kx^2}, \\ a_1^* &= \frac{(\lambda \sigma_\eta^2 + \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2) \gamma^2 \sigma_\varepsilon^2 \sigma_z^2}{x} > 0, \\ a_2^* &= \frac{\lambda^2 \sigma_\eta^2 + \lambda \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2}{x} > 0. \end{aligned} \quad (18)$$

and the equilibrium effort level is

$$e^* = \frac{a_1^* + a_2^*}{k} = \frac{[(1 + \gamma^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2)) \sigma_\varepsilon^2 + 2\lambda \sigma_\eta^2] \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 + \lambda^2 \sigma_\eta^2}{kx} \quad (19)$$

where

$$\begin{aligned} x &= (1 + k\gamma_m \sigma_\eta^2) \lambda^2 \sigma_\eta^2 \\ &\quad + [2(1 + k\gamma_m \sigma_\eta^2) \lambda \sigma_\eta^2 + (1 + k\gamma_m (\sigma_\eta^2 + \sigma_\varepsilon^2)) (\sigma_\eta^2 + \sigma_\varepsilon^2) \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 \\ &\quad + (1 + 2k\lambda \gamma_m \sigma_\eta^2) \sigma_\varepsilon^2] \gamma^2 \sigma_\varepsilon^2 \sigma_z^2, \\ y &= [((1 - \alpha)^2 \sigma_\eta^2 + (1 - k\alpha^2 \gamma_m \sigma_\eta^2) \sigma_\varepsilon^2) \lambda^2 + \alpha^2 \gamma^2 \sigma_\varepsilon^4 \sigma_z^2 (1 - k\gamma_m (\sigma_\eta^2 + \sigma_\varepsilon^2))] \\ &\quad \times [((1 - \alpha)^2 \sigma_\eta^2 + \sigma_\varepsilon^2) \lambda^2 + \alpha^2 \gamma^2 \sigma_\varepsilon^4 \sigma_z^2]. \end{aligned}$$

Both the weight a_1^* on the liquidation value and the weight a_2^* on the stock price are greater than zero as in Holmström and Tirole (1993).¹³ Thus, the manager has an incentive to increase both the liquidation value and the stock price to raise his income. On the other hand, Baiman and Verrecchia (1995) and Kang and Liu (2010) derive an equilibrium contract within which managerial compensation decreases in the liquidation value. They justify this result by arguing that since stock prices impound extra information in addition to information about managerial efforts, executive compensation should be adjusted by the negative weight on the liquidation value.

Similar to the benchmark, in the equilibrium, the individual rationality constraint is binding and the manager's expected utility $\mathbb{E}[u_m^*]$ becomes one. Then, we have

$$\mathbb{E}[I^*] = \frac{\gamma_m}{2} \text{Var}[I^*] + \frac{1}{2} k (e^*)^2 = \frac{e^*}{2}. \quad (20)$$

¹³See also Kim and Suh (1993) and Calcagno and Heider (2014) among others.

Since

$$\mathbb{E}[v^*] = e^*, \quad (21)$$

we have

$$\mathbb{E}[u_o^*] = \delta \mathbb{E}[v^* - I^*] = \frac{\delta e^*}{2}. \quad (22)$$

Under the market-based compensation contract, $\mathbb{E}[I^*]$, $\mathbb{E}[v^*]$ and $\mathbb{E}[u_o^*]$ have positive linear relationship with optimal effort level e^* as in the benchmark. However, (22) shows that as the increase in the expected liquidation value exceeds that in the expected manager's income, the inside owner's expected utility increases in the managerial effort level.

As in the benchmark, the managerial effort level and optimal contract depend on γ_m and k . Theorem 8 shows that as the manager is more risk-averse or his effort becomes costlier, the weights on performance measures a_1^* and a_2^* decrease, while the effect on fixed income a_0^* is not unambiguously determined. Similar to the benchmark, an increase in γ_m decreases managerial effort level e^* indirectly via a_1^* and a_2^* and an increase in k decreases e^* both directly and indirectly via a_1^* and a_2^* from (19). Then, the manager's expected income $\mathbb{E}[I^*]$ decreases from (20) and expected liquidation value $\mathbb{E}[v^*]$ is also reduced. According to (22), we know that the decrease in $\mathbb{E}[v^*]$ dominates that in $\mathbb{E}[I^*]$ and the inside owner's expected utility $\mathbb{E}[u_o^*]$ decreases. As in the benchmark, the inside owner prefers to make a contract with a manager who is less risk-averse and has as high effort efficiency as possible. We summarize the arguments above in the following corollary.

Corollary 9. *If the manager's degree γ_m of risk aversion or the manager's degree k of effort inefficiency increases, the following hold.*

- (1) *Weight a_1^* on the liquidation value and weight a_2^* on the stock price decrease.*
- (2) *The manager's optimal effort level e^* , the manager's expected income $\mathbb{E}[I^*]$, expected liquidation value $\mathbb{E}[v^*]$, and the inside owner's expected utility $\mathbb{E}[u_o^*]$ decrease.*

Now, we compare the manager's choices of effort level and their effects in our model and in the benchmark.

Proposition 10. *The following hold.*

- (1) *The manager's optimal effort level is higher under the market-based compensation contract than in the benchmark (i.e., $e^* > \hat{e}$).*
- (2) *The manager's expected income is higher under the market-based compensation contract than in the benchmark (i.e., $\mathbb{E}[I^*] > \mathbb{E}[\hat{I}]$).*
- (3) *The expected liquidation value is higher under the market-based compensation contract than in the benchmark (i.e., $\mathbb{E}[v^*] > \mathbb{E}[\hat{v}]$).*
- (4) *The inside owner obtains higher expected utility when she offers the market-based compensation contract than in the benchmark (i.e., $\mathbb{E}[u_o^*] > \mathbb{E}[\hat{u}_o]$).*

The first claim shows that the moral hazard problem between the inside owner and manager is relieved by incorporating the stock price into the contract. In the benchmark, the optimal effort level increases in \hat{a}_1 from (15), while it increases in $a_1^* + a_2^*$ in our model from (19). Compared with the benchmark, the weight on the liquidation value is lower since

$$a_1^* - \hat{a}_1 = - \frac{(1 + k\gamma_m \sigma_\eta^2) \lambda^2 \sigma_\eta^2 + [(1 + k\gamma_m \sigma_\eta^2) \lambda \sigma_\eta^2 + (1 + k\lambda \gamma_m \sigma_\eta^2) \sigma_\varepsilon^2] \gamma^2 \sigma_\varepsilon^2 \sigma_z^2}{(1 + k\gamma_m (\sigma_\eta^2 + \sigma_\varepsilon^2)) x} < 0, \quad (23)$$

while the weight on the stock price is higher since $a_2^* > 0$. However, since the increase in the weight on the stock price exceeds the decrease in the weight on the liquidation value, the manager makes more efforts under the market-based compensation contract than in the benchmark. Therefore, if the inside owner offers the market-based compensation contract to the manager, she can expect a higher liquidation value, while she should pay higher expected compensation to the manager than in the benchmark. However, since the increase in the expected liquidation value exceeds that in the manager's expected income, the inside owner can achieve higher expected utility. Indeed, Theorem 8 implies that

the inside owner achieves higher expected utility under the market-based compensation contract than in the benchmark (i.e., $\mathbb{E}[u_o^*] > \mathbb{E}[\hat{u}_o]$). If not, she puts zero weight on the stock price and offers the contract in Proposition 6.¹⁴

Indeed, all claims in Proposition 10 hold only when the manager is risk-averse. Suppose that the manager is risk-neutral (i.e., $\gamma_m = 0$). In the benchmark, the inside owner offers the compensation contract where $\hat{a}_1 = 1$ to the manager and in our model, she offers the contract where

$$a_1^* = \frac{\gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\lambda \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2))}{\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 [2\lambda \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2) + \sigma_\varepsilon^2]}$$

$$a_2^* = \frac{\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\lambda \sigma_\eta^2 + \sigma_\varepsilon^2)}{\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 [2\lambda \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2) + \sigma_\varepsilon^2]}$$

with $a_1^* + a_2^* = 1$ to him. Thus, the manager does not care about the contract scheme when he chooses his effort level (i.e., $\hat{e} = e^* = 1/k$), which implies that the market-based compensation contract cannot relieve the moral hazard problem between the inside owner and manager. Furthermore, the same effort level induces the same expected managerial income and the same expected liquidation value in both models. Therefore, by making a compensation contract with a risk-neutral manager, the inside owner achieves the same expected utility under the market-based compensation contract and in the benchmark. Price informativeness τ_p does not depend on the manager's risk preference. Thus, the stock price contains the same amount of information about managerial efforts regardless of whether the manager is risk-averse or risk-neutral. However, if the manager is risk-neutral, information from the stock price cannot contribute to enhancing the inside owner's expected utility.

4.3. COMPARATIVE STATICS

To analyze the changes in the relative importance of the performance measures in the optimal contract, let us define the relative weight on the liquidation

¹⁴The incentive contract in the benchmark is a special case of the market-based compensation contract, which puts zero weight on the stock price.

value with respect to the stock price as

$$\xi^* \equiv \frac{a_1^*}{a_2^*} = \frac{(\lambda \sigma_\eta^2 + \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2) \gamma^2 \sigma_\varepsilon^2 \sigma_z^2}{\lambda^2 \sigma_\eta^2 + \lambda \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2}. \quad (24)$$

This is independent of the manager's characteristics such as his risk aversion coefficient γ_m and effort inefficiency k . Since $a_1^* > 0$ and $a_2^* > 0$ from Theorem 8, as ξ^* increases (decreases), the contract puts higher weight on liquidation value v (stock price p).

As we have seen in Section 3, the changes in γ , c , σ_z^2 , and σ_η^2 move the proportion λ of informed traders if the information cost has an intermediate value. To analyze these parameters' effects on the optimal contract, we should consider two cases: (i) c takes intermediate values and (ii) c takes extreme values.

4.3.1 Asset Market Equilibrium with Intermediate Information Cost

We consider the case in which information cost c takes an intermediate value such that (8) holds. In the asset market, informed and uninformed traders coexist, i.e., $\lambda \in (0, 1)$, and thus changes in the exogenous parameters generate indirect effects via information acquisition.

Proposition 11. *Suppose that (11) and (14) hold. As rational traders' risk aversion coefficient γ increases satisfying (11) and (14), relative weight ξ^* increases.*

An increase in rational traders' risk aversion has both direct and indirect effects via information acquisition on relative weight ξ^* :

$$\frac{d\xi^*}{d\gamma} = \underbrace{\frac{\partial \xi^*}{\partial \gamma}}_{\text{direct effect of } \gamma} + \underbrace{\frac{\partial \xi^*}{\partial \lambda} \frac{\partial \lambda}{\partial \gamma}}_{\text{indirect effect of } \gamma}.$$

As rational traders are more risk-averse, it follows from (A.29) in the Appendix that γ directly increases ξ^* . Furthermore, the more risk-averse the traders, fewer traders become informed by Proposition 4 when (11) is satisfied, while ξ^* decreases in λ from (A.30) in the Appendix if (14) holds. Thus, the indirect effect

also increases relative weight ξ^* . Since both the direct and indirect effect are positive, an increase in γ makes the liquidation value relatively more important in the contract when rational traders are sufficiently risk-averse and the information cost lies in region C in Proposition 4. The intuition is as follows. According to the second claim of Proposition 5, as γ increases, the stock price contains less information about managerial efforts. However, value informativeness τ_v is not affected by traders' risk preference. It follows that the inside owner puts more weight on the liquidation value relative to the stock price in the contract.

On the other hand, the effects of changes in γ on equilibrium effort level e^* and the inside owner's expected utility $\mathbb{E}[u_o^*]$ are not clear. When (14) holds, an increase in γ directly decreases e^* since

$$\begin{aligned} \frac{\partial e^*}{\partial \gamma} &= -2\gamma_m \gamma \sigma_\varepsilon^4 \sigma_z^2 \\ &\times \frac{\lambda^2 \sigma_\eta^2 ((2\gamma^2 \sigma_\varepsilon^2 \sigma_z^2 - 1) \sigma_\eta^2 + 2(\lambda \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2)) + \gamma^4 \sigma_\varepsilon^4 \sigma_z^4 (\sigma_\eta^2 + \sigma_\varepsilon^2)^2}{x^2} \\ &< 0 \end{aligned}$$

and indirectly increases e^* since $\partial e^*/\partial \lambda < 0$ from (A.31) in the Appendix and $\partial \lambda/\partial \gamma < 0$ from Proposition 4. Therefore, the overall effect on e^* (and $\mathbb{E}[u_o^*]$) is ambiguous.

Proposition 12. *Suppose that (8) and (14) hold. If information cost c increases satisfying (8), the following hold.*

- (1) *Relative weight ξ^* increases.*
- (2) *Managerial effort level e^* , the manager's expected income $\mathbb{E}[I^*]$, expected liquidation value $\mathbb{E}[v^*]$ and the inside owner's expected utility $\mathbb{E}[u_o^*]$ increase.*

Since a_1^* and a_2^* are independent of information cost c , a change in c has no direct effect on relative weight ξ^* , but it indirectly affects ξ^* via information acquisition:

$$\frac{d\xi^*}{dc} = \underbrace{\frac{\partial \xi^*}{\partial \lambda} \frac{\partial \lambda}{\partial c}}_{\text{indirect effect of } c}.$$

An increase in information cost c reduces the proportion λ of informed traders according to the first claim of Proposition 3. On the other hand, if rational traders are sufficiently risk-averse such that (14) holds, as more traders become informed, relative weight ξ^* decreases from (A.30) in the Appendix. Therefore, relative weight ξ^* increases owing to the indirect effect. The intuition is as follows. According to the first claim of Proposition 5, if information about the firm's fundamental value becomes more expensive to acquire, the stock price contains less information about managerial efforts, while value informativeness remains unchanged. Then, the liquidation value becomes relatively more important to the stock price in the contract.

A change in information cost c also has only an indirect effect on the inside owner's expected utility:

$$\frac{de^*}{dc} = \underbrace{\frac{\partial e^*}{\partial \lambda} \frac{\partial \lambda}{\partial c}}_{\text{indirect effect of } c}.$$

As information about θ becomes more expensive to acquire, fewer traders choose to be informed according to the first claim of Proposition 3. Furthermore, as fewer traders become informed, the manager makes more efforts from (A.31) in the Appendix. Thus, an increase in information cost c has the indirect effect of increasing the managerial effort level. From (20), (21), and (22), it follows that the manager's expected income and liquidation value increase and the inside owner becomes better off.

Proposition 13. *Suppose that (8) holds. If market liquidity σ_z^2 increases satisfying (8), the following hold.*

- (1) *Relative weight ξ^* increases.*
- (2) *Managerial effort level e^* , the manager's expected income $\mathbb{E}[I^*]$, expected liquidation value $\mathbb{E}[v^*]$ and the inside owner's expected utility $\mathbb{E}[u_o^*]$ decrease.*

The effect induced by a change in market liquidity σ_z^2 on relative weight ξ^*

is divided into the direct effect and indirect effect via information acquisition:

$$\frac{d\xi^*}{d\sigma_z^2} = \underbrace{\frac{\partial \xi^*}{\partial \sigma_z^2}}_{\text{direct effect of } \sigma_z^2} + \underbrace{\frac{\partial \xi^*}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma_z^2}}_{\text{indirect effect of } \sigma_z^2}. \quad (25)$$

If the stock market becomes more liquid, it directly increases ξ^* since

$$\frac{\partial \xi^*}{\partial \sigma_z^2} = \frac{[\lambda^3 \sigma_\eta^4 + \{2\lambda^2 \sigma_\eta^2 + (\lambda \sigma_\eta^2 + \sigma_\varepsilon^2) \gamma^2 \sigma_\varepsilon^2 \sigma_z^2\} (\sigma_\eta^2 + \sigma_\varepsilon^2) \gamma^2 \sigma_\varepsilon^2 \sigma_z^2] \gamma^2 \sigma_\varepsilon^2}{(\lambda^2 \sigma_\eta^2 + \lambda \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2)^2} > 0.$$

On the other hand, the indirect effect via information acquisition on ξ^* is unclear because ξ^* may increase or decrease in λ while λ always increases in σ_z^2 . However, even when ξ^* decreases owing to the indirect effect, the direct effect dominates the indirect one, which implies that the overall effect on ξ^* is positive. Therefore, relative weight ξ^* always increases in market liquidity σ_z^2 regardless of the sign of the indirect effect. The intuition is as follows. According to the second claim of Proposition 5, as the stock market becomes more liquid, the stock price contains less information about managerial efforts. Meanwhile, value informativeness τ_v is independent of market liquidity. Then, the inside owner offers an incentive contract that puts more weight on the liquidation value relative to the stock price.

A change in market liquidity has a direct effect and an indirect effect via information acquisition on the managerial effort level:

$$\frac{de^*}{d\sigma_z^2} = \underbrace{\frac{\partial e^*}{\partial \sigma_z^2}}_{\text{direct effect of } \sigma_z^2} + \underbrace{\frac{\partial e^*}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma_z^2}}_{\text{indirect effect of } \sigma_z^2}.$$

The managerial effort level may thus increase or decrease because of the direct effect since

$$\frac{\partial e^*}{\partial \sigma_z^2} = - \frac{\gamma_m \gamma^2 \sigma_\varepsilon^4 [\gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2) (2\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2)) - (1 - 2\lambda) \lambda^2 \sigma_\eta^4]}{x^2},$$

whereas the indirect effect is always negative according the second claim of Proposition 3 and (A.31) in the Appendix. However, since the indirect effect

dominates the direct one even when the direct effect is positive, the manager always makes less efforts as the stock market becomes more liquid. Then, the manager's expected income and the expected liquidation value decrease and the inside owner becomes worse off from (20), (21) and (22).

Proposition 14. *Suppose that (8) holds. If variance σ_η^2 of the firm's fundamental value increases satisfying (8), the following hold.*

- (1) *When (14) holds, relative weight ξ^* decreases.*
- (2) *Managerial effort level e^* , the manager's expected income $\mathbb{E}[I^*]$, expected liquidation value $\mathbb{E}[v^*]$ and the inside owner's expected utility $\mathbb{E}[u_o^*]$ decrease.*

The effect of σ_η^2 on relative weight ξ^* is also divided into the direct effect and indirect effect via information acquisition:

$$\frac{d\xi^*}{d\sigma_\eta^2} = \underbrace{\frac{\partial \xi^*}{\partial \sigma_\eta^2}}_{\text{direct effect of } \sigma_\eta^2} + \underbrace{\frac{\partial \xi^*}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma_\eta^2}}_{\text{indirect effect of } \sigma_\eta^2}. \quad (26)$$

The direct effect is positive since

$$\frac{\partial \xi^*}{\partial \sigma_\eta^2} = \frac{(1-\lambda)(\lambda + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2) \gamma^4 \sigma_\varepsilon^6 \sigma_z^4}{(\lambda^2 \sigma_\eta^2 + \lambda \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2)^2} > 0,$$

while the indirect effect is negative according to the second claim of Proposition 3 and (A.30) in the Appendix if traders are sufficiently risk-averse such that (14) is satisfied. However, the former effect is dominated by the latter and thus ξ^* decreases. A change in σ_η^2 affects not only price informativeness τ_p but also value informativeness τ_v . Indeed, the effect of σ_η^2 on price informativeness is not clear, while an increase in σ_η^2 always makes the liquidation value less informative. However, it turns out that the effect induced by the decrease in value informativeness dominates that induced by the decrease in price informativeness.

A change in the variance of the firm's fundamental value also has a direct effect and an indirect effect via information acquisition on managerial effort level:

$$\frac{de^*}{d\sigma_\eta^2} = \underbrace{\frac{\partial e^*}{\partial \sigma_\eta^2}}_{\text{direct effect of } \sigma_\eta^2} + \underbrace{\frac{\partial e^*}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma_\eta^2}}_{\text{indirect effect of } \sigma_\eta^2}.$$

The direct effect negatively affects e^* by (A.32) in Appendix and the indirect effect also negatively affects e^* from (A.31) in the Appendix and the second claim of Proposition 3. Therefore, as the firm's fundamental value becomes more volatile, the manager makes less efforts, which implies a decrease in his expected income, the expected liquidation value, and the inside owner's expected utility from (20), (21), and (22).

4.3.2 Asset Market Equilibrium with an Extreme Information Cost

We now consider extreme cases in which information cost is sufficiently low such that (7) holds and where it is sufficiently high such that (6) holds. According to Proposition 2, all rational traders purchase information about the firm's fundamental value in the former case and no one becomes informed in the latter case. Then, a slight change in information cost c does not affect the proportion of informed traders. Specifically, if (7) holds initially and c moves but keeps holding (7), then λ remains unchanged at one. In addition, if (6) holds initially and c moves but keeps holding (6), then λ remains unchanged at zero. Thus, a change in the exogenous parameters does not have an indirect effect via information acquisition on the contract.

Proposition 15. *Suppose that information cost c is sufficiently low or sufficiently high such that (7) or (6) holds. If information cost c changes satisfying (7) or (6), then ξ^* , e^* , $\mathbb{E}[I^*]$, $\mathbb{E}[v^*]$ and $\mathbb{E}[u_o^*]$ remain unchanged.*

Proposition 15 shows that, if acquiring information is so costly that no one can obtain it, or if information is so inexpensive that it is accessible to everyone,

the inside owner and the manager do not care about a slight change in information cost since a_1^* and a_2^* are not affected by c . A change in the information cost deserves consideration in the contract only when it takes an intermediate value such that (8) is satisfied.

Extremely Low Information Cost

Suppose that the information cost is sufficiently low such that (7) holds. Then, all rational traders choose to purchase information about the firm's fundamental value at cost c (i.e., $\lambda = 1$).

Proposition 16. *Suppose that information cost c is sufficiently low such that (7) holds. The following hold.*

- (1) *If rational traders' degree γ of risk aversion increases satisfying (7), then relative weight ξ^* increases, but e^* , $\mathbb{E}[I^*]$, $\mathbb{E}[v^*]$, and $\mathbb{E}[u_o^*]$ decrease.*
- (2) *If market liquidity σ_z^2 increases satisfying (7), then ξ^* increases, but e^* , $\mathbb{E}[I^*]$, $\mathbb{E}[v^*]$, and $\mathbb{E}[u_o^*]$ decrease.*
- (3) *If variance σ_η^2 of the firm's fundamental value increases satisfying (7), then ξ^* remains unchanged, but e^* , $\mathbb{E}[I^*]$, $\mathbb{E}[v^*]$, and $\mathbb{E}[u_o^*]$ decrease.*

From (A.29) in the Appendix, we know that an increase in γ always directly increases ξ^* . Since there is no indirect effect when the information cost is sufficiently low, as rational traders become more risk-averse, the relative importance of the liquidation value rises in the contract. When the information cost takes an extremely low value, an increase in γ makes the stock price transmit less information about managerial efforts since

$$\left. \frac{\partial \tau_p}{\partial \gamma} \right|_{\lambda=1} = -\frac{2\gamma\sigma_\varepsilon^4\sigma_z^2}{(\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)^2} < 0.$$

Owing to the decrease in the stock price's monitoring performance, the inside owner offers a contract that puts more weight on the liquidation value relative to the stock price. An increase in γ directly decreases e^* from (A.33) in the

Appendix, and thus the manager's expected income and the expected liquidation value decrease and the inside owner becomes worse off from (20), (21), and (22).

As market liquidity increases, relative weight ξ^* increases because of the direct effect from (25). Indeed, if σ_z^2 increases, the stock price becomes less informative since

$$\left. \frac{\partial \tau_p}{\partial \sigma_z^2} \right|_{\lambda=1} = -\frac{\gamma^2 \sigma_\varepsilon^2}{(\sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2)^2} < 0,$$

while τ_v remains unchanged. Consequently, the inside owner and manager agree on a contract that puts more weight on the liquidation value relative to the stock price. The managerial effort level is also directly affected by a change in market liquidity. An increase in market liquidity reduces the managerial effort level from (A.34) in the Appendix and thus the manager's expected income and the liquidation value decrease and the inside owner becomes worse off from (20), (21), and (22).

A change in the volatility of the firm's fundamental value has no effect on relative weight ξ^* since

$$\left. \frac{\partial \xi^*}{\partial \sigma_\eta^2} \right|_{\lambda=1} = 0.$$

As σ_η^2 increases, the stock price contains less information about managerial efforts because

$$\left. \frac{\partial \tau_p}{\partial \sigma_\eta^2} \right|_{\lambda=1} = -\frac{1}{(\sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2)^2} < 0$$

and the liquidation value also becomes less informative according to the fourth claim of Proposition 5. However, it turns out that both effects exactly offset and thus there is no change in relative weight ξ^* in the contract. On the other hand, e^* decreases from (A.35) in the Appendix, and thus $\mathbb{E}[I^*]$, $\mathbb{E}[v^*]$, and $\mathbb{E}[u_o^*]$ decrease from (20), (21), and (22).

Extremely High Information Cost

Now, suppose that the information cost is sufficiently high such that (6) holds. Then, all rational traders refuse to purchase information about the firm's fundamental value at cost c and remain uninformed (i.e., $\lambda = 0$).

Proposition 17. *Suppose that information cost c is sufficiently high such that (6) holds.*

- (1) *If rational traders' degree γ of risk aversion increases satisfying (6), then relative weight ξ^* increases, but e^* , $\mathbb{E}[I^*]$, $\mathbb{E}[v^*]$, and $\mathbb{E}[u_o^*]$ decrease.*
- (2) *If market liquidity σ_z^2 increases satisfying (6), then ξ^* increases, but e^* , $\mathbb{E}[I^*]$, $\mathbb{E}[v^*]$, and $\mathbb{E}[u_o^*]$ decrease.*
- (3) *If variance σ_η^2 of the firm's fundamental value increases satisfying (6), then ξ^* increases, but e^* , $\mathbb{E}[I^*]$, $\mathbb{E}[v^*]$, and $\mathbb{E}[u_o^*]$ decrease.*

Since an increase in γ always directly increases ξ^* from (A.29) in the Appendix, as rational traders become more risk-averse, relative weight ξ^* increases. We have

$$\left. \frac{\partial \tau_p}{\partial \gamma} \right|_{\lambda=0} = -\frac{2}{\gamma^3 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2)^2} < 0.$$

This makes the inside owner offer a contract with a higher weight on the liquidation value. Managerial effort level e^* decreases from (A.36) in the Appendix, and thus e^* , $\mathbb{E}[I^*]$, $\mathbb{E}[v^*]$, and $\mathbb{E}[u_o^*]$ decrease from (20), (21), and (22).

As the stock market becomes more liquid, relative weight ξ^* increases because of the direct effect from (25). As σ_z^2 increases, the stock price contains less information about managerial efforts since

$$\left. \frac{\partial \tau_p}{\partial \sigma_z^2} \right|_{\lambda=0} = -\frac{1}{\gamma^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2)^2} < 0.$$

Then, the inside owner offers a contract in which a higher weight is put on the liquidation value. An increase in market liquidity directly decreases the manager's effort level from (A.37) in the Appendix. This finding implies that the manager expects lower income, the firm expects a lower liquidation value, and the inside owner becomes worse off from (20), (21), and (22).

An increase in σ_η^2 directly increases relative weight ξ^* from (26). In the absence of the indirect effect, the inside owner always puts more weight on the liquidation value as the firm's fundamental value becomes more volatile. The stock price has less information as σ_η^2 increases since

$$\left. \frac{\partial \tau_p}{\partial \sigma_\eta^2} \right|_{\lambda=0} = -\frac{2}{\gamma^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2)^3} < 0$$

and the liquidation value also contains less information according to the fourth claim of Proposition 5. However, it turns out that a decrease in price informativeness dominates that in value informativeness and the liquidation value becomes more important in the contract than the stock price. On the other hand, as σ_η^2 increases, the manager makes less efforts from (A.38) in the Appendix, and thus $\mathbb{E}[I^*]$, $\mathbb{E}[v^*]$, and $\mathbb{E}[u_\phi^*]$ decrease from (20), (21), and (22).

So far, we characterize our equilibrium compensation contract. Since information cost cannot generate the direct effect, it affects on the contract only when the cost has intermediate value. In this case, if rational traders are sufficiently risk averse, as information cost becomes more expensive, the liquidation value becomes more important in the contract. An increase in market liquidity increases the importance of the liquidation value in the contract whether information cost has intermediate value or it has extreme values. In other words, the existence of the indirect effect cannot change the direction of the effects generated by market liquidity on the contract. The effect of the variance of the firm's fundamental value provides us interesting results. The directions of the effect made by a change in σ_η^2 depend on information cost. When rational traders are sufficiently risk averse, if information cost has intermediate value, the overall effect increases the importance of the stock price in the contract. On the contrary, if information cost is extremely high, an increase in σ_η^2 makes the firm's liquidation value more important by the indirect effect. On the other hand, if information cost is extremely low, both the direct and indirect effects become zero and thus the relative importance is not affected by a change in σ_η^2 .

5. WELFARE ANALYSIS

In this section, we examine the welfare effect induced by incorporating the stock price into the compensation contract. Similar to Kang and Liu (2010), we measure social welfare as the sum of the ex ante expected utilities of the inside owner,

manager, and rational traders.¹⁵ In the benchmark, social welfare is given by

$$S\hat{W} = \mathbb{E}[\hat{u}_o(\hat{v} - \hat{I})] + \mathbb{E}\left[u_m\left(\hat{I} - \frac{1}{2}k\hat{e}^2\right)\right] + \lambda\mathbb{E}[u(\hat{w}_i)] + (1 - \lambda)\mathbb{E}[u(\hat{w}_u)]$$

and in our model, it is given by

$$SW^* = \mathbb{E}[u_o^*(v^* - I^*)] + \mathbb{E}\left[u_m\left(I^* - \frac{1}{2}k(e^*)^2\right)\right] + \lambda\mathbb{E}[u(w_i^*)] + (1 - \lambda)\mathbb{E}[u(w_u^*)].$$

Then, we have the following result.

Proposition 18. *Social welfare is higher under the market-based compensation contract than the benchmark (i.e., $SW^* > S\hat{W}$).*

From (3), we know that the ex ante expected utilities of rational traders are independent of the compensation contract. Furthermore, a change in the stock price due to managerial effort is exactly offset by a change in the liquidation value and thus their utilities remain unchanged. Therefore, the ex ante expected utilities of rational traders are equivalent regardless of whether the stock price is incorporated into the contract. The ex ante expected utility of the manager is also independent of the compensation scheme since the individual rationality constraints make his expected utility equal to his reservation utility. On the contrary, as we have seen in Subsection 4.2, the inside owner can expect higher utility by incorporating the stock price into the compensation contract than otherwise. Consequently, she can raise social welfare by offering a market-based compensation contract.

6. CONCLUDING REMARKS

This study investigates market-based compensation contracts when endogenous information acquisition is allowed. To do so, we incorporate the standard principal–agent problem into Grossman and Stiglitz’s (1980) asset market model. We show

¹⁵We ignore the expected utility of noise traders since they are not utility maximizers.

that the inside owner can increase her utility by offering a market-based compensation contract to the manager. Compared with a contract excluding the stock price, the manager chooses a higher effort level under the market-based compensation contract, which raises both the expected liquidation value and the expected executive compensation. However, since the former's increase dominates the latter's, the inside owner can obtain additional utility. Indeed, the increase in the manager's effort level is related to the manager's risk preference. If the manager is risk-neutral, his effort level is not affected by the contract scheme and thus the inside owner's expected utility remains unchanged regardless of whether she offers a market-based compensation contract.

We also examine how the optimal contract is affected by exogenous parameters such as information cost, market liquidity, and the volatility of the firm's fundamental value. If the information cost in the asset market takes an intermediate value, the optimal contract is affected by both the indirect effects via the information market as well as the direct effects. On the contrary, if the information cost is sufficiently high or low, it only directly influences the contract. We then analyze the effects of the exogenous parameters on the optimal contract from the perspectives of price informativeness and value informativeness.

Finally, this study shows that incorporating the stock price into the contract helps raise social welfare, defined as the sum of the ex ante expected utilities of the inside owner, manager, and rational traders. This is because the ex ante utilities of the manager and rational traders are not affected by the contract scheme, while the inside owner can increase her expected utility by offering a market-based compensation contract to the manager.

Future research could consider ambiguous information about the firm's future value. Practically, there is plenty of information in financial markets and thus the inside owner and outside traders may have difficulty in estimating the quality of the observed information, while the manager has relatively precise information. Therefore, we could construct a model in which the inside owner and outside traders have multiple beliefs about the firm's fundamental value.

. APPENDIX

PROOF OF PROPOSITION 1: We conjecture that P is a strictly increasing function of s . Then, information about θ contained in P is equivalent to that in s . There are two cases to consider: (i) $\lambda = 0$ and (ii) $\lambda \in (0, 1]$.

(i) Suppose that $\lambda = 0$. Then, the market-clearing condition (1) becomes $x_u = 1 - \delta - z$. Since $P(s)$ is not correlated with v , the market-clearing condition implies that

$$P(s) = e^* + \gamma(\sigma_\eta^2 + \sigma_\varepsilon^2).$$

(ii) Suppose that $\lambda \in (0, 1]$. Since s and v are normally distributed, we have

$$\begin{aligned} \mathbb{E}[\tilde{v}|P(s) = p] &= \mathbb{E}[\tilde{v}|\tilde{s} = s] = \frac{\gamma^2 \sigma_\varepsilon^4 \sigma_z^2 e + \lambda^2 \sigma_\eta^2 s}{\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2}, \\ \text{Var}[\tilde{v}|P(s) = p] &= \text{Var}[\tilde{v}|\tilde{s} = s] = \frac{\sigma_\varepsilon^2 (\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2)}{\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2}. \end{aligned}$$

From the market-clearing condition (1), we obtain

$$P(s) = (1 - \alpha)e^* + \alpha s$$

where

$$\alpha = \frac{\lambda^2 \sigma_\eta^2 + \lambda \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \lambda \gamma^2 \sigma_\varepsilon^4 \sigma_z^2}{\lambda^2 \sigma_\eta^2 + \lambda \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2}. \quad \blacksquare$$

PROOF OF PROPOSITION 2: Note that φ is an increasing function of λ from (5).

- (1) If (6) holds, then $\varphi(0) \geq 1$ and $\varphi(\lambda) > 1$ for every $\lambda \in (0, 1]$. Therefore, no rational traders become informed, i.e., $\lambda = 0$. \square
- (2) If (7) holds, then $\varphi(\lambda) < 1$ for every $\lambda \in [0, 1)$ and $\varphi(1) \leq 1$. Therefore, all rational traders become informed, i.e., $\lambda = 1$. \square

(3) If (8) holds, we have

$$\begin{aligned}\varphi(0) &= e^{\gamma c} \sqrt{\frac{\sigma_\varepsilon^2}{\sigma_\eta^2 + \sigma_\varepsilon^2}} < e^{\gamma \bar{c}} \sqrt{\frac{\sigma_\varepsilon^2}{\sigma_\eta^2 + \sigma_\varepsilon^2}} = 1, \\ \varphi(1) &= e^{\gamma c} \sqrt{\frac{\sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2}{\sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2)}} \\ &> e^{\gamma \bar{c}} \sqrt{\frac{\sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2}{\sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2)}} = 1.\end{aligned}$$

Thus, there exists $\lambda \in (0, 1)$ such that $\varphi(\lambda) = 1$. ■

PROOF OF PROPOSITION 3: Suppose that (8) holds.

- (1) From (4), we observe that when c increases, κ should decrease to satisfy $\varphi(\lambda) = 1$. Since κ is an increasing function of λ from (5), λ should decrease when c increases to satisfy $\varphi(\lambda) = 1$. □
- (2) To satisfy $\varphi(\lambda) = 1$ when σ_η^2 or σ_z^2 moves, κ should remain unchanged. Thus, ζ_z in (10) should remain unchanged when σ_z^2 moves and ζ_η in (9) should do when σ_η^2 moves. Since ζ_z (ζ_η) increases in λ and decreases in σ_z^2 (σ_η^2), an increase in σ_z^2 (σ_η^2) leads to an increase in λ to keep κ unchanged. ■

PROOF OF PROPOSITION 4: Suppose that (11) holds. Since $\varphi(\lambda) = 1$ in the equilibrium, we have

$$\frac{\lambda^2}{\sigma_\varepsilon^2 \sigma_z^2} = \gamma^2 \left(\frac{1}{e^{2\gamma c} - 1} - \frac{\sigma_\varepsilon^2}{\sigma_\eta^2} \right).$$

Let

$$\zeta_\gamma \equiv \gamma^2 \left(\frac{1}{e^{2\gamma c} - 1} - \frac{\sigma_\varepsilon^2}{\sigma_\eta^2} \right).$$

Thus, if ζ_γ is a decreasing function of γ , as γ increases, λ decreases to satisfy $\varphi(\lambda) = 1$. We need to check the sign of

$$\frac{\partial \zeta_\gamma}{\partial \gamma} = - \frac{2\gamma(\sigma_\varepsilon^2 m^2 - (1 - \gamma c)\sigma_\eta^2 m + \gamma c \sigma_\eta^2)}{\sigma_\eta^2 m^2},$$

where

$$m = e^{2\gamma c} - 1 > 0.$$

We have two cases to consider:

(i) If $c \geq 1/\gamma$, then $\partial \zeta_\gamma / \partial \gamma < 0$ trivially holds.

(ii) If $c < 1/\gamma$, then ζ_γ decreases in γ for every m when

$$(1 - \gamma c)^2 \sigma_\eta^2 - 4\gamma c \sigma_\varepsilon^2 < 0,$$

which is equivalent to

$$c \in \left(\frac{\sigma_\varepsilon^2}{\gamma \sigma_\eta^2} \left[\sqrt{1 + \frac{\sigma_\eta^2}{\sigma_\varepsilon^2}} - 1 \right]^2, \frac{\sigma_\varepsilon^2}{\gamma \sigma_\eta^2} \left[\sqrt{1 + \frac{\sigma_\eta^2}{\sigma_\varepsilon^2}} + 1 \right]^2 \right).$$

Since

$$\frac{1}{\gamma} < \frac{\sigma_\varepsilon^2}{\gamma \sigma_\eta^2} \left(\sqrt{1 + \frac{\sigma_\eta^2}{\sigma_\varepsilon^2}} + 1 \right)^2,$$

if (11) holds, an increase in γ leads to a decrease in λ . ■

PROOF OF PROPOSITION 5:

(1) Suppose that (14) holds. We have

$$\frac{d\tau_p}{dc} = \frac{\partial \tau_p}{\partial \lambda} \frac{\partial \lambda}{\partial c}.$$

Since

$$\frac{\partial \tau_p}{\partial \lambda} = \frac{2\lambda \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^4 \sigma_z^2 \psi_1}{\alpha [(\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2)(\lambda \sigma_\eta^2 + \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2)]^2} > 0, \quad (\text{A.27})$$

where

$$\psi_1 = \lambda \sigma_\eta^2 (\gamma^2 \sigma_\varepsilon^2 \sigma_z^2 + \lambda^2 - \lambda) + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2 (\lambda + \gamma^2 \sigma_\eta^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 - 1) > 0$$

and $\partial \lambda / \partial c < 0$ according to the first claim of Proposition 3, τ_p decreases in c . □

(2) Suppose that (11) and (14) hold. We have

$$\frac{d\tau_p}{d\gamma} = \frac{\partial\tau_p}{\partial\gamma} + \frac{\partial\tau_p}{\partial\lambda} \frac{\partial\lambda}{\partial\gamma}.$$

Since

$$\frac{\partial\tau_p}{\partial\gamma} = -\frac{2\lambda\gamma\sigma_\varepsilon^4\sigma_z^2\psi_2}{\alpha(\lambda\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^2\sigma_z^2(\sigma_\eta^2 + \sigma_\varepsilon^2))^2(\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)^2} < 0, \quad (\text{A.28})$$

where

$$\begin{aligned} \psi_2 &= \lambda\sigma_\eta^4(\gamma^4\sigma_\varepsilon^4\sigma_z^4 - \lambda^2 + 2\lambda^3) \\ &\quad + \gamma^2\sigma_\varepsilon^2\sigma_z^2[\lambda\sigma_\eta^2((\gamma^2\sigma_\varepsilon^2\sigma_z^2 - 1)\sigma_\varepsilon^2d + \lambda(2\sigma_\eta^2 + 3\sigma_\varepsilon^2)) \\ &\quad + \gamma^2\sigma_\varepsilon^4\sigma_z^2(\sigma_\eta^2 + \sigma_\varepsilon^2)] > 0, \end{aligned}$$

an increase in γ directly decreases τ_p . Note that $\partial\tau_p/\partial\lambda > 0$ from (A.27) and $\partial\lambda/\partial\gamma < 0$ from Proposition 4. Thus, an increase in γ indirectly decreases τ_p . Therefore, τ_p decreases in γ . \square

(3) Since we have

$$\begin{aligned} \frac{d\tau_p}{d\sigma_z^2} &= \frac{\partial\tau_p}{\partial\sigma_z^2} + \frac{\partial\tau_p}{\partial\lambda} \frac{\partial\lambda}{\partial\sigma_z^2} \\ &= -\frac{\lambda\gamma^2\sigma_\varepsilon^4(\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^2\sigma_z^2(\sigma_\eta^2 + \sigma_\varepsilon^2))}{\alpha(\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)(\lambda\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^2\sigma_z^2(\sigma_\eta^2 + \sigma_\varepsilon^2))^2} < 0, \end{aligned}$$

τ_p decreases in σ_z^2 . \square

(4) This is straightforward from (13). \blacksquare

PROOF OF PROPOSITION 6: In the equilibrium, we have

$$\begin{aligned} \mathbb{E}[\hat{I}] &= \hat{a}_0 + \hat{a}_1\hat{e}, \\ \text{Var}[\hat{I}] &= \hat{a}_1^2(\sigma_\eta^2 + \sigma_\varepsilon^2). \end{aligned}$$

The first-order condition for the manager's incentive compatibility implies $\hat{e} = \hat{a}_1/k$. Since the individual rationality condition for the manager holds as equality in the equilibrium, we see that

$$\mathbb{E}[\hat{v} - \hat{I}] = \frac{\hat{a}_1}{k} - \frac{\hat{a}_1^2(\sigma_\eta^2 + \sigma_\varepsilon^2)\gamma_m}{2} - \frac{\hat{a}_1^2}{2k}.$$

The first-order condition for the owner is given by

$$\frac{1 - \hat{a}_1 [1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2)]}{k} = 0,$$

which implies

$$\hat{a}_1 = \frac{1}{1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2)}.$$

Therefore,

$$\hat{e} = \frac{1}{k(1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))}.$$

From the individual rationality condition, we have

$$\mathbb{E}[\hat{I}] = \hat{a}_0 + \hat{a}_1 \mathbb{E}[\hat{v}] = \frac{\gamma_m}{2} \text{Var}[\hat{I}] + \frac{1}{2} k \hat{e}^2,$$

which implies that

$$\hat{a}_0 = \frac{-1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2)}{2k(1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))^2}. \quad \blacksquare$$

PROOF OF THEOREM 8: Since we have different price functions when $\lambda = 0$ and $\lambda \in (0, 1]$ from Proposition 1, we consider two cases: (i) $c \geq \bar{c}$ and (ii) $c < \bar{c}$.

(i) Suppose that $c \geq \bar{c}$. From the second claim of Proposition 2, we know $\lambda = 0$. From Proposition 1, we have

$$\begin{aligned} \mathbb{E}[\bar{I}] &= \bar{a}_0 + \bar{a}_1 \bar{e} + \bar{a}_2 [\bar{e} - (1 - \delta)(\sigma_\eta^2 + \sigma_\varepsilon^2)\gamma] \\ \text{Var}[\bar{I}] &= \bar{a}_1^2(\sigma_\eta^2 + \sigma_\varepsilon^2) + \bar{a}_2^2 \gamma^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2)^2. \end{aligned}$$

There is no correlation between v and p when $\lambda = 0$. The first-order condition for the manager's incentive compatibility implies

$$\bar{e} = \frac{\bar{a}_1 + \bar{a}_2}{k}.$$

Since the individual rationality condition for the manager holds as equality in the equilibrium, we have

$$\mathbb{E}[\bar{v} - \bar{I}] = \frac{\bar{a}_1 + \bar{a}_2}{k} - \frac{\gamma_m}{2} \left[\bar{a}_1^2(\sigma_\eta^2 + \sigma_\varepsilon^2) + \bar{a}_2^2 \gamma^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2)^2 \right] - \frac{(\bar{a}_1 + \bar{a}_2)^2}{2k}.$$

The first-order conditions for the owner are given by

$$\begin{aligned}\frac{1 - (1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))\bar{a}_1 - \bar{a}_2}{k} &= 0, \\ \frac{1 - \bar{a}_1 - (1 + k\gamma_m\gamma^2\sigma_z^2(\sigma_\eta^2 + \sigma_\varepsilon^2)^2)\bar{a}_2}{k} &= 0,\end{aligned}$$

which implies

$$\begin{aligned}\bar{a}_1 &= \frac{\gamma^2\sigma_z^2(\sigma_\eta^2 + \sigma_\varepsilon^2)}{1 + (1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))(\sigma_\eta^2 + \sigma_\varepsilon^2)\gamma^2\sigma_z^2}, \\ \bar{a}_2 &= \frac{1}{1 + (1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))(\sigma_\eta^2 + \sigma_\varepsilon^2)\gamma^2\sigma_z^2}.\end{aligned}$$

Therefore,

$$\bar{e} = \frac{1 + \gamma^2\sigma_z^2(\sigma_\eta^2 + \sigma_\varepsilon^2)}{k[1 + (1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))(\sigma_\eta^2 + \sigma_\varepsilon^2)\gamma^2\sigma_z^2]}.$$

From the individual rationality condition, we have

$$\mathbb{E}[\bar{I}] = \bar{a}_0 + \bar{a}_1\mathbb{E}[\bar{v}] + \bar{a}_2\mathbb{E}[\bar{p}] = \frac{\gamma_m}{2}\text{Var}[\bar{I}] + \frac{1}{2}k\bar{e}^2,$$

which implies

$$\bar{a}_0 = -\frac{y_0 - 2(1 - \delta)(\sigma_\eta^2 + \sigma_\varepsilon^2)^3 k^2 \gamma_m \gamma^3 \sigma_z^2}{2[1 + (1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))(\sigma_\eta^2 + \sigma_\varepsilon^2)\gamma^2\sigma_z^2]^2 k}.$$

(ii) From the first and third claims of Proposition 2, we know $\lambda \in (0, 1]$. From Proposition 1, we have

$$\begin{aligned}\mathbb{E}[I^*] &= a_0^* + a_1^*e^* + \left(e^* - \frac{\gamma\sigma_\varepsilon^2}{\lambda}(1 - \delta)\right)a_2^*, \\ \text{Var}[I^*] &= (a_1^*)^2(\sigma_\eta^2 + \sigma_\varepsilon^2) + (a_2^*)^2\alpha^2\left(\sigma_\eta^2 + \frac{\gamma^2\sigma_\varepsilon^4\sigma_z^2}{\lambda^2}\right) + 2a_1^*a_2^*\alpha\sigma_\eta^2.\end{aligned}$$

The first-order condition for the manager's incentive compatibility implies

$$e^* = \frac{a_1^* + a_2^*}{k}.$$

Since the individual rationality condition for the manager holds as equality in the equilibrium, we have

$$\begin{aligned} \mathbb{E}[v^* - I^*] &= \frac{a_1^* + a_2^*}{k} - \frac{\gamma_m}{2} [(a_1^*)^2(\sigma_\eta^2 + \sigma_\varepsilon^2) \\ &\quad + (a_2^*)^2\alpha^2 \left(\sigma_\eta^2 + \frac{\gamma^2 \sigma_\varepsilon^4 \sigma_z^2}{\lambda^2} \right) + 2a_1^* a_2^* \alpha \sigma_\eta^2] - \frac{(a_1^* + a_2^*)^2}{2k}. \end{aligned}$$

The first-order conditions for the owner are given by

$$\begin{aligned} \frac{1 - (1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))a_1^* - (1 + k\alpha\gamma_m\sigma_\eta^2)a_2^*}{k} &= 0, \\ \frac{1 - a_1^* - a_2^*}{k} - \alpha\gamma_m \left(a_1^* \sigma_\eta^2 + a_2^* \alpha \left(\sigma_\eta^2 + \frac{\gamma^2 \sigma_\varepsilon^4 \sigma_z^2}{\lambda^2} \right) \right) &= 0, \end{aligned}$$

which implies

$$\begin{aligned} a_1^* &= \frac{(\lambda \sigma_\eta^2 + \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2) \gamma^2 \sigma_\varepsilon^2 \sigma_z^2}{x} > 0, \\ a_2^* &= \frac{\lambda^2 \sigma_\eta^2 + \lambda \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2}{x} > 0. \end{aligned}$$

Therefore,

$$e^* = \frac{[(1 + \gamma^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2)) \sigma_\varepsilon^2 + 2\lambda \sigma_\eta^2] \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 + \lambda^2 \sigma_\eta^2}{kx}.$$

From the individual rationality condition, we have

$$\mathbb{E}[I^*] = a_0^* + a_1^* \mathbb{E}[\bar{v}] + a_2^* \mathbb{E}[\bar{p}] = \frac{\gamma_m}{2} \text{Var}[\bar{I}] + \frac{1}{2} k (e^*)^2,$$

which implies

$$a_0^* = - \frac{y + 2(1 - \delta) ((1 - \alpha) \sigma_\eta^2 + \sigma_\varepsilon^2) kx \lambda \gamma \sigma_\varepsilon^2}{2kx^2}.$$

Since $\lim_{\lambda \rightarrow 0} a_0^* = \bar{a}_0$, $\lim_{\lambda \rightarrow 0} a_1^* = \bar{a}_1$, and $\lim_{\lambda \rightarrow 0} a_2^* = \bar{a}_2$, for every $\lambda \in [0, 1]$, we have (18) and (19). \blacksquare

PROOF OF PROPOSITION 10:

(1) We have

$$\begin{aligned} e^* - \hat{e} &= \frac{a_1^* + a_2^* - \hat{a}_1}{k} \\ &= \frac{(\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2) \gamma_m \sigma_\varepsilon^2}{(1 + k \gamma_m (\sigma_\eta^2 + \sigma_\varepsilon^2)) x} > 0. \end{aligned}$$

Thus, the manager chooses a higher effort level under the market-based compensation contract than the benchmark. \square

(2) From (16) and (20), we have

$$\mathbb{E}[I^*] - \mathbb{E}[\hat{I}] = \frac{e^* - \hat{e}}{2},$$

which is greater than zero according to the first claim. \blacksquare

PROOF OF PROPOSITION 11: We have

$$\frac{d\xi^*}{d\gamma} = \frac{\partial \xi^*}{\partial \gamma} + \frac{\partial \xi^*}{\partial \lambda} \frac{\partial \lambda}{\partial \gamma}.$$

Since

$$\frac{\partial \xi^*}{\partial \gamma} = \frac{2\gamma \sigma_\varepsilon^2 \sigma_z^2 [\lambda^3 \sigma_\eta^4 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2) (2\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\lambda \sigma_\eta^2 + \sigma_\varepsilon^2))]}{(\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\lambda \sigma_\eta^2 + \sigma_\varepsilon^2))^2} > 0, \quad (\text{A.29})$$

risk aversion coefficient γ directly increases ξ^* . If (14) holds, then

$$\begin{aligned} \frac{\partial \xi^*}{\partial \lambda} &= -\frac{1}{(\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\lambda \sigma_\eta^2 + \sigma_\varepsilon^2))^2} \\ &\quad \times \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2 [\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 ((\gamma^2 \sigma_\varepsilon^2 \sigma_z^2 - 1) \sigma_\varepsilon^2 + 2\lambda (\sigma_\eta^2 + \sigma_\varepsilon^2) \\ &\quad + \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2 \sigma_z^2)] < 0 \end{aligned} \quad (\text{A.30})$$

and if (11) holds, then $\partial \lambda / \partial \gamma < 0$ from Proposition 4. Then, an increase in γ directly increases ξ^* . Therefore, ξ^* increases in γ . \blacksquare

PROOF OF PROPOSITION 12:

(1) We have

$$\frac{d\xi^*}{dc} = \frac{\partial \xi^*}{\partial \lambda} \frac{\partial \lambda}{\partial c}.$$

If (14) holds, then $\partial \xi^*/\partial \lambda < 0$ from (A.30), and if (8) holds, then $\partial \lambda/\partial c < 0$ according to the first claim of Proposition 3. Therefore, ξ^* increases in c . \square

(2) Suppose that (8) holds. We have

$$\frac{de^*}{dc} = \frac{\partial e^*}{\partial \lambda} \frac{\partial \lambda}{\partial c}.$$

Since

$$\frac{\partial e^*}{\partial \lambda} = -\frac{2\gamma_m \gamma^2 \sigma_\eta^2 \sigma_\varepsilon^4 (1-\lambda) [\lambda \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2)]}{x^2} < 0 \quad (\text{A.31})$$

and $\partial \lambda/\partial c < 0$ according to the first claim of Proposition 3, we have $de^*/dc > 0$. Therefore, $\mathbb{E}[u_o^*]$ increases in c according to the second claim of Corollary 9. \blacksquare

PROOF OF PROPOSITION 13:

(1) Suppose that (8) holds. We have

$$\begin{aligned} \frac{d\xi^*}{d\sigma_z^2} &= \frac{\partial \xi^*}{\partial \sigma_z^2} + \frac{\partial \xi^*}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma_z^2} \\ &= \frac{\gamma^2 \sigma_\varepsilon^2}{2(\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\lambda \sigma_\eta^2 + \sigma_\varepsilon^2))^2} \\ &\quad \times [\lambda^3 \sigma_\eta^4 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 ((2\lambda (\sigma_\eta^2 + \sigma_\varepsilon^2) + \sigma_\varepsilon^2) \lambda \sigma_\eta^2 + \\ &\quad \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2) (2\sigma_\varepsilon^2 + \lambda \sigma_\eta^2))] > 0. \end{aligned}$$

Therefore, ξ^* increases in σ_z^2 . \square

(2) Since

$$\begin{aligned} \frac{de^*}{d\sigma_z^2} &= \frac{\partial e^*}{\partial \sigma_z^2} + \frac{\partial e^*}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma_z^2} \\ &= -\frac{\gamma_m \gamma^2 \sigma_\varepsilon^4}{x^2} \\ &\quad \times (\lambda \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2)) (\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2)) \\ &< 0, \end{aligned}$$

e^* decreases in σ_z^2 . Therefore, $\mathbb{E}[u_o^*]$ decreases in σ_z^2 according to the second claim of Corollary 9. ■

PROOF OF PROPOSITION 14: Suppose that (8) holds.

(1) If (14) holds,

$$\begin{aligned} \frac{d\xi^*}{d\sigma_\eta^2} &= \frac{\partial \xi^*}{\partial \sigma_\eta^2} + \frac{\partial \xi^*}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma_\eta^2} \\ &= -\frac{\gamma^4 \sigma_\varepsilon^6 \sigma_z^4}{2\lambda \sigma_\eta^2 (\lambda^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\lambda \sigma_\eta^2 + \sigma_\varepsilon^2))^2} \\ &\quad \times [\lambda^2 \sigma_\eta^2 (2\lambda + 2\gamma^2 \sigma_\varepsilon^2 \sigma_z^2 - 1) \\ &\quad + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2 (2\lambda + \gamma^2 \sigma_\eta^2 \sigma_z^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 - 1)] \\ &< 0. \end{aligned}$$

Thus, ξ^* decreases in σ_η^2 . □

(2) We have

$$\frac{de^*}{d\sigma_\eta^2} = \frac{\partial e^*}{\partial \sigma_\eta^2} + \frac{\partial e^*}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma_\eta^2}.$$

Since

$$\frac{\partial e^*}{\partial \sigma_\eta^2} = -\frac{\gamma_m (\lambda + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2) \psi_3}{x^2} < 0, \quad (\text{A.32})$$

where

$$\begin{aligned} \psi_3 &= \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 [\gamma^2 \sigma_\varepsilon^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_\varepsilon^2) (3\lambda \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^4 \sigma_z^2 \\ &\quad + (2 - \lambda + \gamma^2 \sigma_\eta^2 \sigma_z^2) \sigma_\varepsilon^2) + \lambda \sigma_\eta^2 (3\lambda \sigma_\eta^2 + 2\sigma_\varepsilon^2)] \\ &\quad + \lambda^3 \sigma_\eta^4 \\ &> 0, \end{aligned}$$

and $\partial e^* / \partial \lambda < 0$ from (A.31) and $\partial \lambda / \partial \sigma_\eta^2 > 0$ according to the second claim of Proposition 3, we have $de^* / d\sigma_\eta^2 < 0$. Therefore, $\mathbb{E}[u_o^*]$ decreases in σ_η^2 from Corollary 9. ■

PROOF OF PROPOSITION 16: Suppose that (7) holds.

(1) Since we have

$$\left. \frac{\partial \xi^*}{\partial \gamma} \right|_{\lambda=1} = 2\gamma\sigma_\varepsilon^2\sigma_z^2 > 0,$$

ξ^* increases in γ when $\lambda = 1$. Since

$$\left. \frac{\partial e^*}{\partial \gamma} \right|_{\lambda=1} = -\frac{2\gamma_m\gamma\sigma_\varepsilon^4\sigma_z^2}{[1+k\gamma_m\sigma_\eta^2+\gamma^2\sigma_\varepsilon^2\sigma_z^2(1+k\gamma_m(\sigma_\eta^2+\sigma_\varepsilon^2))]^2} < 0, \quad (\text{A.33})$$

e^* decreases in γ when $\lambda = 1$, which implies that $\mathbb{E}[u_o^*]$ decreases in γ when $\lambda = 1$ according to the second claim of Corollary 9. \square

(2) Since we have

$$\left. \frac{\partial \xi^*}{\partial \sigma_z^2} \right|_{\lambda=1} = \gamma^2\sigma_\varepsilon^2 > 0,$$

relative weight ξ^* increases in σ_z^2 when $\lambda = 1$. Since

$$\left. \frac{\partial e^*}{\partial \sigma_z^2} \right|_{\lambda=1} = -\frac{\gamma_m\gamma^2\sigma_\varepsilon^4}{[1+k\gamma_m\sigma_\eta^2+\gamma^2\sigma_\varepsilon^2\sigma_z^2(1+k\gamma_m(\sigma_\eta^2+\sigma_\varepsilon^2))]^2} < 0, \quad (\text{A.34})$$

e^* decreases in σ_z^2 when $\lambda = 1$, which implies that $\mathbb{E}[u_o^*]$ decreases in σ_z^2 when $\lambda = 1$ according to the second claim of Corollary 9. \square

(3) Since we have

$$\left. \frac{\partial \xi^*}{\partial \sigma_\eta^2} \right|_{\lambda=1} = 0,$$

relative weight ξ^* is not affected by a change in σ_η^2 when $\lambda = 1$. Since

$$\left. \frac{\partial e^*}{\partial \sigma_\eta^2} \right|_{\lambda=1} = -\frac{\gamma_m(1+\gamma^2\sigma_\varepsilon^2\sigma_z^2)^2}{[1+k\gamma_m\sigma_\eta^2+\gamma^2\sigma_\varepsilon^2\sigma_z^2(1+k\gamma_m(\sigma_\eta^2+\sigma_\varepsilon^2))]^2} < 0, \quad (\text{A.35})$$

e^* decreases in σ_η^2 when $\lambda = 1$, which implies that $\mathbb{E}[u_o^*]$ decreases in σ_η^2 when $\lambda = 1$ according to the second claim of Corollary 9. \blacksquare

PROOF OF PROPOSITION 17: Suppose that (6) holds.

(1) Since we have

$$\left. \frac{\partial \xi^*}{\partial \gamma} \right|_{\lambda=0} = 2\gamma\sigma_z^2(\sigma_\eta^2 + \sigma_\varepsilon^2) > 0,$$

ξ^* increases in γ when $\lambda = 0$. Since

$$\left. \frac{\partial e^*}{\partial \gamma} \right|_{\lambda=0} = -\frac{2\gamma_m\gamma\sigma_z^2(\sigma_\eta^2 + \sigma_\varepsilon^2)^2}{[1 + \gamma^2\sigma_z^2(\sigma_\eta^2 + \sigma_\varepsilon^2)(1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))]^2} < 0, \quad (\text{A.36})$$

e^* decreases in γ when $\lambda = 0$, which implies that $\mathbb{E}[u_o^*]$ decreases in γ when $\lambda = 0$ according to the second claim of Corollary 9. \square

(2) Since we have

$$\left. \frac{\partial \xi^*}{\partial \sigma_z^2} \right|_{\lambda=0} = \gamma^2(\sigma_\eta^2 + \sigma_\varepsilon^2) > 0,$$

relative weight ξ^* increases in σ_z^2 when $\lambda = 0$. Since

$$\left. \frac{\partial e^*}{\partial \sigma_z^2} \right|_{\lambda=0} = -\frac{\gamma_m\gamma^2(\sigma_\eta^2 + \sigma_\varepsilon^2)^2}{[1 + \gamma^2\sigma_z^2(\sigma_\eta^2 + \sigma_\varepsilon^2)(1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))]^2} < 0, \quad (\text{A.37})$$

e^* decreases in σ_z^2 when $\lambda = 0$, which implies that $\mathbb{E}[u_o]$ decreases in σ_z^2 when $\lambda = 0$ according to the second claim of Corollary 9. \square

(3) Since we have

$$\left. \frac{\partial \xi^*}{\partial \sigma_\eta^2} \right|_{\lambda=0} = \gamma^2\sigma_z^2 > 0,$$

relative weight ξ^* increases in σ_η^2 when $\lambda = 0$. Since

$$\left. \frac{\partial e^*}{\partial \sigma_\eta^2} \right|_{\lambda=0} = -\frac{\gamma_m\gamma^2\sigma_z^2(\sigma_\eta^2 + \sigma_\varepsilon^2)(2 + \gamma^2\sigma_z^2(\sigma_\eta^2 + \sigma_\varepsilon^2))}{[1 + \gamma^2\sigma_z^2(\sigma_\eta^2 + \sigma_\varepsilon^2)(1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))]^2} < 0, \quad (\text{A.38})$$

e^* decreases in σ_η^2 when $\lambda = 0$, which implies that $\mathbb{E}[u_o]$ decreases in σ_η^2 when $\lambda = 0$ according to the second claim of Corollary 9. \blacksquare

PROOF OF PROPOSITION 18: Since the individual rationality constraints are binding in equilibrium contracts in both our model and the benchmark, the manager's expected utility in each model is given by

$$\mathbb{E} \left[u_m \left(\hat{I} - \frac{1}{2}k\hat{e}^2 \right) \right] = \mathbb{E} \left[u_m \left(I^* - \frac{1}{2}k(e^*)^2 \right) \right] = 1.$$

From (3), the expected utilities of rational traders are not affected by the contract variables. The expected utility of the inside owner is higher in our model than in the benchmark according to the fourth claim of Proposition 10. Therefore, $SW^* > S\hat{W}$. ■

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